1 Rational Numbers

INTRODUCTION

Natural Numbers

The counting numbers 1, 2, 3, 4, ... are called *natural numbers*.

The smallest natural number is 1 but there is no largest, because regardless of how large a number is chosen, there exist larger ones. We can always add 1 to any natural number to get its successor. Thus, we can say that there are infinitely many natural numbers. The collection of natural numbers is denoted by **N**.

Thus, $N = \{1, 2, 3, 4, ...\}$

Properties of Natural Numbers

(*i*) The sum of two natural numbers is again a natural number. [Closure Property of Addition]

(*ii*) *Addition of natural numbers is commutative.*

If a and b are any two natural numbers, then

$$a + b = b + a$$

(*iii*) *Addition of natural numbers is associative.*

If a, b and c are any three natural numbers, then

$$(a + b) + c = a + (b + c)$$

(*iv*) The product of two natural numbers is again a natural number. [Closure Property of Multiplication]

(*v*) *Multiplication of natural numbers is commutative.*

If a and b are any two natural numbers, then

$$a \times b = b \times a$$

(*vi*) *Multiplication of natural numbers is associative.*

If a, b and c are any three natural numbers, then

$$(a \times b) \times c = a \times (b \times c)$$

(*vii*) *1 is called the multiplicative identity for natural numbers.*

If a is any natural number, then $1 \cdot a = a \cdot 1 = a$

(*viii*) *Multiplication of natural numbers is distributive over addition.* [Distributive Property]

If a, b and c are any three natural numbers, then

$$a \times (b + c) = a \times b + a \times c$$

Whole Numbers

Consider a simple equation $x + 5 = 11$ ...(1)

The solution of this equation is $x = 11 - 5$, *i.e.*, $x = 6$ and 6 is a natural number. $x = 6$ is the solution of the equation $x + 5 = 11$

Now, consider another equation $x + 3 = 3$ ...(2)

The solution of this equation is $x = 3 - 3 \Rightarrow x = 0$.

So, we cannot solve this equation, if we consider only natural numbers. There is no natural number which satisfies the equation (2). There is a need to extend the number system further.

In the set of natural numbers, if we include the number 0, the resulting set is known as the set of whole numbers. It is represented by **W**.

Thus, $W = \{0, 1, 2, 3, ...\}$

To solve equation like (2), we added the number zero (0) to the collection of natural numbers and obtained numbers {0, 1, 2, 3,.....}

Properties of Whole Numbers

(*i*) *Closure property of addition.*

The sum of two whole numbers is again a whole number or, whole numbers are closed for addition.

(*ii*) *Addition of whole numbers is commutative.*

If *a* and *b* are any two whole numbers, then

$$a + b = b + a$$

(*iii*) *Addition of whole numbers is associative.*

If *a*, *b*, *c* are any three whole numbers, then

$$(a + b) + c = a + (b + c)$$

(*iv*) *The sum of any whole number and 0 is the whole number itself.*

If *a* is any whole number, then

$$a + 0 = 0 + a = a$$

0 is called the *identity element of addition* (or *additive identity*) of whole numbers.

(*v*) *The product of two whole numbers is again a whole number.* [Closure property of multiplication]

(*vi*) *Multiplication of whole numbers is commutative.*

If *a* and *b* are any two whole numbers, then

$$a \times b = b \times a$$

(*vii*) *Multiplication of whole numbers is associative.*

If *a*, *b*, *c* are any three whole numbers, then

$$(a \times b) \times c = a \times (b \times c)$$

(*viii*) *The product of 1 and any whole number is the whole number itself.*

If *a* is any whole number, then

$$a \cdot 1 = 1 \cdot a = a$$

This is called the multiplication property of 1.

1 is called the *identity element for multiplication* (or *the multiplicative identity*) of whole numbers.

(*ix*) *Multiplication of whole numbers is distributive over addition.* [Distributive property]

If *a*, *b*, *c* are any three whole numbers, then

$$a \times (b + c) = a \times b + a \times c$$

Integers

Now, consider an equation $x + 5 = 3$

The solution of this equation is $x = 3 - 5$, *i.e.*, $x = -2$.

But, –2 is not a whole number. Thus, there is a need to extend the number system further.

The whole numbers together with the opposites of the counting numbers, or negative numbers i.e., {..., –4, –3, –2, –1} make up the set of numbers, which we call the collection of **integers**. The set of integers is denoted by Z.

Thus, Z = {... , –4, –3, –2, –1, 0, 1, 2, 3, 4, ... }

'Z' comes from the German word "zahlen" which means "to count".

Referred to the set I, 1, 2, 3, 4, ... are called **positive integers** and –1, –2, –3, –4, ... are called **negative integers.**

The number 0 is neither positive nor negative.

Properties of Integers

(i) *Closure property of addition.*

The set of integers is closed under the operation of addition because the sum of any two integers is always another integer.

(ii) *Addition of integers is commutative.*

If a and b are any two integers, then

$$a + b = b + a$$

(iii) *Addition of integers is associative.*

If a, b, c are any three integers, then

$$(a + b) + c = a + (b + c)$$

(iv) *The sum of any integer and zero is the integer itself.*

If a is any integer, then $a + 0 = 0 + a = a$

0 is called the *identity element of addition* (or *additive identity*) of integers.

(v) *For each non-zero integer 'a', there is an integer '– a' such that*

$$a + (-a) = (-a) + a = 0$$

'$-a$' is called the *negative* (or *additive inverse*) of a.

(vi) *The product of two integers is always an integer.* [Closure property of multiplication]

(vii) *Multiplication of integers is commutative.*

If a and b are any two integers, then

$$a \times b = b \times a$$

(viii) *Multiplication of integers is associative.*

If a, b, c are any three integers, then

$$(a \times b) \times c = a \times (b \times c)$$

(ix) *The product of zero and any integer is always zero* (0). [Multiplication property of 0]

If a is an integer, then $a \times 0 = 0$

(x) *The product of one and any integer is the integer itself.* [Multiplication property of 1]

If a is an integer, then $a \times 1 = 1 \times a = a$

1 is called the *multiplicative identity* for integers.

(xi) *Multiplication of integers is distributive over addition and subtraction.* [Distributive property]

If a, b, c are any three integers, then

$$a \times (b + c) = a \times b + a \times c$$

and $$a \times (b - c) = a \times b - a \times c$$

RATIONAL NUMBERS

Consider the equation $2x = 15$ ...(4)

Does the solution exist in N, W or Z? No, there is no such number in these collections which will satisfy the equation.

We need the number $\frac{15}{2}$ to solve the equation. Thus, there is a need to extend the number system further.

A number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a *rational number*. The set of rational numbers is denoted by Q. Thus $Q = \left\{\frac{1}{3}, \frac{-12}{13}, 19, \frac{9}{14}, etc.\right\}$

A rational number may be positive, negative or zero.

The rational number $\frac{p}{q}$ is positive if p and q have like signs, and negative, if p and q have unlike signs.

Hence, the set of numbers of the form $\frac{p}{q}$, includes zero, positive and negative integers, positive and negative fractions. All such numbers are called rational numbers.

Numbers $\frac{3}{5}, \frac{5}{2}, -\frac{2}{9}, -2, 0, 4, 1, \frac{1}{3}$, etc. are rational numbers.

Remember

1. A rational number is a number that can be expressed as a fraction $\frac{p}{q}$, where p and q are integers and $q \neq 0$. p is the numerator and q is the denominator of rational number $\frac{p}{q}$. For example, in a rational number $\frac{3}{7}$, 3 is the numerator and 7 is the denominator.
2. A fraction is a part of the whole or a collection. Fractions are always positive. A rational number can be positive, negative or zero. Every fraction is a rational number but every rational number is not a fraction. For example, $-\frac{2}{5}$ is rational number but not a fraction.

Positive Rational Numbers

A rational number which has both the numerator and the denominator with same sign, either positive or negative is a positive rational number.

For example, $\frac{1}{3}, \frac{2}{5}, \frac{-5}{-11}, \frac{-7}{-17}$ are positive rational numbers as their numerator and denominator have the same sign.

Negative Rational Numbers

A rational number with either the numerator or the denominator negative is a negative rational number.

For example, $\frac{3}{-5}, \frac{-4}{7}, \frac{-13}{19}, \frac{17}{-23}$ are negative rational numbers as only the numerator or the denominator is negative.

Note that even 0, which can be written as $\frac{0}{1}$, is a rational number.

Standard Form of a Rational Number

A rational number $\frac{p}{q}$ is said to be in standard form if q is positive and the integers p and q have no common divisor other than 1, *i.e.* HCF of p and q = 1 or p and q are co-primes.

For example, consider $\frac{-12}{-21}$. Denominator of the rational number $\frac{-12}{-21}$ is –21, which is negative.

Multiplying the numerator and denominator of $\frac{-12}{-21}$ by –1, we get

$$\frac{-12}{-21} = \frac{(-12)\times(-1)}{(-21)\times(-1)} = \frac{12}{21}$$

Greatest common divisor of 12 and 21 is 3.

Now, $$\frac{12}{21} = \frac{12 \div 3}{21 \div 3} = \frac{4}{7}$$

Hence, the standard form of a rational number $\frac{-12}{-21}$ is $\frac{4}{7}$.

Lowest Form or Simplest Form of a Rational Number

A rational number $\frac{p}{q}$, $q \neq 0$ is said to be in the lowest form if its numerator p and denominator q have no common factor except 1, *i.e.*, if p and q are relatively prime to each other. For example,

(*i*) $\frac{-9}{21} = \frac{-3}{7}$ (Lowest form) (*ii*) $\frac{-35}{-56} = \frac{5}{8}$ (Lowest form) (*iii*) $\frac{-7}{17}$ (Lowest form)

Equivalent Rational Numbers

Equivalent rational numbers have the same value.

If $\frac{p}{q}$ is a given rational number and k is a non-zero integer, then $\frac{p}{q} = \frac{kp}{kq}$. Here, $\frac{p}{q}$ and $\frac{kp}{kq}$ are called equivalent rational numbers.

For example, $\frac{2}{3} = \frac{2\times2}{3\times2} = \frac{4}{6}$. Also, $\frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15}$.

Therefore, $\frac{2}{3}, \frac{4}{6}$ and $\frac{10}{15}$ are equivalent rational numbers.

Again, if $\frac{p}{q}$ is a given rational number and m is a common divisor of p and q, then $\frac{p}{q} = \frac{p \div m}{q \div m}$

For example, $\frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4}$

Also, $\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$ and $\frac{27}{36} = \frac{27 \div 9}{36 \div 9} = \frac{3}{4}$

Therefore, $\frac{15}{20}, \frac{18}{24}, \frac{27}{36}, \frac{3}{4}$ are equivalent rational numbers.

Equality of Rational Numbers (Using Cross Multiplication)

Two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal, *i.e.*, $\frac{a}{b} = \frac{c}{d}$ iff $a \times d = b \times c$

$\Rightarrow$ Numerator of first × Denominator of second = Denominator of first × Numerator of second

For example, $\frac{5}{7} = \frac{15}{21}$ only if $5 \times 21 = 7 \times 15$.

Now, $5 \times 21 = 105$ and $7 \times 15 = 105$. As $5 \times 21 = 7 \times 15$ (each product = 105), so $\frac{5}{7} = \frac{15}{21}$.

Remember

1. The set of rational numbers is denoted by Q and includes all positive numbers, negative numbers and zero that can be written as a ratio (fraction) of one number over another.
2. A rational number is positive when both numerator and denominator have same sign, else negative. In other words, a rational number is negative, if its numerator and denominator are of the opposite signs. That is negative rational numbers can be written as a fraction that have a value less than zero.
3. A rational number $\frac{a}{b}$ is said to be in the standard form if b is positive, and the integers a and b have no common divisor other than 1.
4. A rational number $\frac{a}{b}$ is said to be in the lowest form or simplest form if a and b have no common factor other than 1.
5. Equivalent rational numbers are those numbers which have same value but are represented in different ways.

For example, $\frac{2}{3} = \frac{2\times2}{3\times2} = \frac{2\times3}{3\times3} = \frac{2\times4}{3\times4} = \frac{2\times5}{3\times5}$ that is $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$ all are equivalent rational numbers. We can write infinite rational numbers equivalent to a given rational number. Each of the equivalent fractions have same value.

6. Let $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} = \frac{c}{d}$ only when $ad = bc$ and $\frac{a}{b} \neq \frac{c}{d}$ when $ad \neq bc$.

Solved Examples

1. *Define rational number. Give at least six examples of such numbers.*

Sol. A number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number.

Examples: $\frac{3}{8}, \frac{-5}{7}, \frac{2}{5}, \frac{-2}{9}, \frac{11}{17}, \frac{-19}{56}, \frac{21}{31}$ are all rational numbers.

2. *Express* $\frac{2}{7}$ *as a rational number with denominator as* 21, 35 *and* 49.

Sol. To convert a rational number $\frac{2}{7}$ with denominator 21, we multiply the denominator and numerator by 3.

$$\frac{2}{7} = \frac{2\times3}{7\times3} = \frac{6}{21}$$

Similarly, $\frac{2}{7} = \frac{2\times5}{7\times5} = \frac{10}{35}$ and $\frac{2}{7} = \frac{2\times7}{7\times7} = \frac{14}{49}$

3. *Which of the following rational numbers are positive or negative?*

$$\frac{-4}{5}, \frac{11}{-31}, \frac{7}{9}, \frac{-13}{-17}, \frac{-28}{-17}, \frac{93}{118}, \frac{29}{-81}$$

Sol. $\frac{7}{9}, \frac{-13}{-17}, \frac{-28}{-17}, \frac{93}{118}$ are positive rational numbers as their numerators and denominators have the same sign.

$\frac{-4}{5}, \frac{11}{-31}, \frac{29}{-81}$ are negative rational numbers as only the numerator or the denominator is negative, that is, its numerator and denominator have opposite sign.

4. *State whether the following statements are true (T) or false (F).*

(*i*) *If* $\frac{r}{s}$ *is a rational number, then s cannot be equal to zero.*

(*ii*) *Every fraction is a rational number.*

(*iii*) 0 *is a whole number but it is not a rational number.*

(*iv*) *Every integer is a rational number.*

Sol. (*i*) As fractions with zero denominators often lead to meaningless solution because dividing by zero is an invalid operation.

So, if $\frac{r}{s}$ is a rational number, then s cannot be equal to zero, is a true statement.

(*ii*) Any number that can be written in fraction form is a rational number. A rational number is a number that can be expressed as a fraction of two integers p and q. Hence, the given statement, every fraction is a rational number, is a true statement.

(*iii*) The given statement is false because 0 is a whole number and also 0 is a rational number.

(*iv*) Every integer is a rational number, since each integer x can be written as $\frac{x}{1}$. For example, –3 is an integer and $-3 = \frac{-3}{1}$ and thus –3 is a rational number because $\frac{-3}{1}$ is a fraction whose numerator and denominator are integers. Hence, the given statement, every integer is a rational number, is a true statement.

5. *Fill in the blanks to make the statement true:*

(*i*) *The equivalent rational number of* $\frac{5}{7}$, *whose numerator is 45 is* ....

(*ii*) *The equivalent rational number of* $\frac{7}{9}$, *whose denominator is 45 is* ....

(*iii*) $\frac{3}{4}$ *and* $\frac{9}{12}$ *are* .... *fractions.*

(*iv*) *The lowest form of rational number* $\frac{18}{30}$ *is* ....

(*v*) *The lowest form of* $\frac{-13}{26}$ *is* ....

Sol. (*i*) $\frac{5}{7}=\frac{5\times 9}{7\times 9}=\frac{45}{63}$

$\therefore$ The equivalent rational number of $\frac{5}{7}$, whose numerator is 45 is $\frac{45}{63}$

(*ii*) $\frac{7}{9}=\frac{7\times 5}{9\times 5}=\frac{35}{45}$

$\therefore$ The equivalent rational number of $\frac{7}{9}$, whose denominator is 45 is $\frac{35}{45}$

(*iii*) $\frac{9}{12}=\frac{9\div 3}{12\div 3}=\frac{3}{4}$ [$\because$ HCF of 9 and 12 is 3]

$\therefore$ $\frac{3}{4}$ and $\frac{9}{12}$ are equivalent fractions.

(*iv*) $\frac{18}{30}=\frac{18\div 6}{30\div 6}=\frac{3}{5}$ [$\because$ HCF of 18 and 30 is 6]

$\therefore$ The lowest form of rational number $\frac{18}{30}$ is $\frac{3}{5}$.

(*v*) $\frac{-13}{26}=\frac{(-13)\div 13}{26\div 13}=\frac{-1}{2}$ [$\because$ HCF of 13 and 26 is 13]

$\therefore$ The lowest form of $\frac{-13}{26}$ is $\frac{-1}{2}$.

6. *Select the rational numbers from the list which are also the integers.*

$$\frac{8}{4}, \frac{9}{3}, \frac{7}{4}, \frac{6}{4}, \frac{5}{3}, \frac{8}{3}, \frac{7}{3}, \frac{6}{3}, \frac{5}{3}, \frac{4}{2}, \frac{3}{1}, \frac{3}{2}, \frac{1}{1}, \frac{0}{1}, \frac{-1}{1}, \frac{-2}{1}, \frac{-3}{2}, \frac{-4}{2}, \frac{-5}{2}, \frac{-6}{2}$$

Sol. In the given list of numbers, there are rational numbers which are not integers and there are rational numbers which are integers.

$$\frac{8}{4}=\frac{8\div 4}{4\div 4}=\frac{2}{1}=2$$

[Dividing numerator and denominator by common divisor 4]

So, $\frac{8}{4}=2$, which is an integer.

Similarly,

$$\frac{9}{3}=\frac{9\div 3}{3\div 3}=\frac{3}{1}=3;\ \frac{6}{3}=\frac{6\div 3}{3\div 3}=\frac{2}{1}=2$$

$$\frac{4}{2}=\frac{4\div 2}{2\div 2}=\frac{2}{1}=2;\ \frac{3}{1}=3;\ \frac{1}{1}=1;\ \frac{0}{1}=0$$

$$\frac{-1}{1}=-1;\ \frac{-2}{1}=-2;\ \frac{-4}{2}=\frac{-4\div 2}{2\div 2}=\frac{-2}{1}=-2;$$

$$\frac{-6}{2}=\frac{-6\div 2}{2\div 2}=\frac{-3}{1}=-3$$

Hence, $\frac{8}{4}, \frac{9}{3}, \frac{6}{3}, \frac{4}{2}, \frac{3}{1}, \frac{1}{1}, \frac{0}{1}, \frac{-1}{1}, \frac{-2}{1}, \frac{-4}{2}, \frac{-6}{2}$ are rational numbers which are also integers.

7. *Select those which can be written as a rational number with denominator 4.*

$$\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, \frac{-16}{11}, \frac{5}{-4}, \frac{140}{28}$$

Sol. $\frac{64}{16} = \frac{64 \div 4}{16 \div 4} = \frac{16}{4}$; $\quad \frac{36}{-12} = \frac{36 \div (-3)}{-12 \div (-3)} = \frac{-12}{4}$

$\frac{5}{-4} = \frac{5 \times (-1)}{-4 \times (-1)} = \frac{-5}{4}$; $\quad \frac{140}{28} = \frac{140 \div 7}{28 \div 7} = \frac{20}{4}$

Hence, $\frac{64}{16}, \frac{36}{-12}, \frac{5}{-4}, \frac{140}{28}$ can be written as a rational number with denominator 4.

Miscellaneous Solved Examples

There are four options (Q1 and Q2) out of which only one is correct. Choose the correct option.

1. *A number which can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is*

(*a*) *natural number* (*b*) *whole number* (*c*) *integer* (*d*) *rational number*

Sol. A number which can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is rational number.

∴ (*d*) is the correct answer.

2. *A number of the form $\frac{p}{q}$ is said to be a rational number if*

(*a*) *p and q are integers* (*b*) *p and q are integers and $q \neq 0$*

(*c*) *p and q are integers and $p \neq 0$* (*d*) *p and q are integers and $p \neq 0$ and $q \neq 0$*

Sol. A number of the form $\frac{p}{q}$ is said to be a rational number if p and q are integers and $q \neq 0$.

∴ (*b*) is the correct answer.

3. *Fill in the blanks to make the statements true.*

(*i*) *The equivalent rational number of $\frac{3}{5}$, whose numerator is 33 is ....*

(*ii*) *The equivalent rational number of $\frac{5}{9}$, whose denominator is 36 is ....*

Sol. (*i*) $\frac{3}{5} = \frac{3 \times 11}{5 \times 11} = \frac{33}{55}$

[Multiplying numerator by 11 and therefore denominator by 11 to get 33 as numerator]

∴ The equivalent rational number of $\frac{3}{5}$ whose numerator is 33 is $\frac{33}{55}$.

(*ii*) $\frac{5}{9} = \frac{5 \times 4}{9 \times 4} = \frac{20}{36}$

∴ The equivalent rational number of $\frac{5}{9}$ whose denominator is 36 is $\frac{20}{36}$.

4. *In each of the following, state whether the statements are true (T) or false (F).*

(*i*) *If $\frac{x}{y}$ is a rational number, then y is always a whole number.*

(*ii*) *If $\frac{p}{q}$ is a rational number, then p cannot be equal to zero.*

(*iii*) *If $\frac{r}{s}$ is a rational number, then s cannot be equal to zero.*

Sol. (*i*) If $\frac{x}{y}$ is a rational number, then x and y are integers and $y \neq 0$.

So, the given statement is false (F).

(*ii*) If $\frac{p}{q}$ is a rational number, p, q are integers and $q \neq 0$, so p can be equal to zero because 0 is an integer.

Hence, the given statement is false (F).

(*iii*) If $\frac{r}{s}$ is a rational number, then r, s are integers and $s \neq 0$.

Hence, the given statement is true (T).

5. *Pick out the positive rational numbers from amongst the following:*

$$\frac{2}{3}, \frac{-2}{3}, \frac{2}{-3}, \frac{-2}{-3}$$

Sol. We know that a rational number $\frac{p}{q}$ is said to be positive if, p and q have like signs, and negative if, p and q have unlike signs.

So, the positive rational numbers are $\frac{2}{3}$ and $\frac{-2}{-3}$.

6. *Explain why we do not include numbers, like* $\frac{5}{0}, \frac{-19}{0}, \frac{-3}{0}$, *where zero is in the denominators in rational numbers.*

Sol. We know that division by zero (0) is not possible, so we do not include numbers like $\frac{5}{0}, \frac{-19}{0}, \frac{-3}{0}$, in rational numbers. A rational number is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

7. *When is a rational number said to be in the standard form? Write the following rational numbers in their standard forms:* $\frac{6}{14}, \frac{10}{140}, \frac{-9}{3}, \frac{3}{-7}, \frac{-5}{-9}$.

Sol. A rational number in the lowest form is said to be in the standard form, if its denominator is positive.

The standard form of $\frac{6}{14} = \frac{6 \div 2}{14 \div 2} = \frac{3}{7}$

The standard form of $\frac{10}{140} = \frac{10 \div 10}{140 \div 10} = \frac{1}{14}$

The standard form of $\frac{-9}{3} = \frac{-9 \div 3}{3 \div 3} = \frac{-3}{1}$

The standard form of $\frac{3}{-7} = \frac{3 \div (-1)}{-7 \div (-1)} = \frac{-3}{7}$

The standard form of $\frac{-5}{-9} = \frac{-5 \div (-1)}{-9 \div (-1)} = \frac{5}{9}$

8. *Pick out the rational number from the following list which is not equal to* $\frac{3}{5}$.

$$\frac{-3}{5}, \frac{-3}{-5}, \frac{3}{5}, \frac{6}{10}, \frac{-12}{-20}, \frac{30}{50}.$$

Sol.

$$\frac{-3}{-5} = \frac{-3 \div (-1)}{-5 \div (-1)} = \frac{3}{5}, \quad \frac{3}{5} = \frac{3}{5}, \quad \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

$$\frac{-12}{-20} = \frac{-12 \div (-4)}{-20 \div (-4)} = \frac{3}{5}, \quad \frac{30}{50} = \frac{30 \div 10}{50 \div 10} = \frac{3}{5}$$

But, $\frac{-3}{5} \neq \frac{3}{5}$.

Hence, the rational number which is not equal to $\frac{3}{5}$ is $\frac{-3}{5}$.

9. *Write* $\frac{2}{9}$ *in five different forms. For example,* $\frac{-4}{-18}$.

Sol. We know that a rational number may be replaced by an equivalent rational number $\frac{p \times k}{q \times k}$, where k is any positive or negative integer.

So,
$$\frac{2}{9} = \frac{2\times2}{9\times2} = \frac{4}{18}, \quad \frac{2}{9} = \frac{2\times5}{9\times5} = \frac{10}{45},$$
$$\frac{2}{9} = \frac{2\times8}{9\times8} = \frac{16}{72}, \quad \frac{2}{9} = \frac{2\times(-3)}{9\times(-3)} = \frac{-6}{-27}$$
$$\frac{2}{9} = \frac{2\times(-7)}{9\times(-7)} = \frac{-14}{-63}$$

Exercise 1.1

1. What is a rational number? Give five examples of rational numbers.
2. What are positive and negative rational numbers? Give three examples of each.
3. (*i*) Which of the following rational numbers are positive and which are negative?
$$\frac{7}{9}, \frac{-2}{9}, \frac{-7}{15}, 8, \frac{-5}{-13}, \frac{3}{-5}$$
   (*ii*) Is every natural number a positive rational number?
   (*iii*) Is every rational number a whole number?
4. What is the standard form of a rational number? Give five examples.
5. (*i*) Express $\frac{-195}{-180}$ in standard form.
   (*ii*) Express $\frac{-77}{-99}$ in standard form.
6. (*i*) What do you understand by lowest form of a rational number? Give some examples.
   (*ii*) Reduce: $\frac{15}{33}$ and $\frac{28}{49}$ to lowest terms.
7. What do you understand by equality of rational numbers? Give some examples.
8. What are equivalent rational numbers? Give some examples.
9. Fill in the blanks:
   (*i*) Standard form of -1 is ....
   (*ii*) Every fraction is a .... number.
   (*iii*) $\frac{-8}{20}$ and $\frac{12}{-30}$ are .... rational numbers.
   (*iv*) $\frac{-4}{7}, \frac{8}{-14}, \frac{-12}{21}$ are .... rational numbers.

ADDITION AND SUBTRACTION OF RATIONAL NUMBERS

1. When the rational numbers have same denominators

When we add or subtract rational numbers that have the same denominator, we add or subtract only the numerators. The denominators stay the same.

Let $\frac{p}{q}$ and $\frac{r}{q}$ be two given rational numbers then $\frac{p}{q} + \frac{r}{q} = \frac{p+r}{q}$ and $\frac{p}{q} - \frac{r}{q} = \frac{p-r}{q}$

$\therefore$ Sum of $\frac{p}{q}$ and $\frac{r}{q} = \frac{p+r}{q}$ and their difference $= \frac{p-r}{q}$

For example, (*i*) $\frac{1}{7} + \frac{3}{7} = \frac{1+3}{7} = \frac{4}{7}$ (*ii*) $\frac{-6}{11} + \frac{4}{11} = \frac{(-6)+4}{11} = \frac{-2}{11}$ (*iii*) $\frac{8}{11} - \frac{5}{11} = \frac{8-5}{11} = \frac{3}{11}$